Paper Reference(s)

6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced

Tuesday 23 June 2009 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP3), the paper reference (6669), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

M35145A

1. Solve the equation

$$7 \operatorname{sech} x - \tanh x = 5$$

Give your answers in the form $\ln a$, where a is a rational number.

(5)

2.

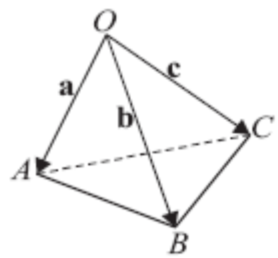

Figure 1

The points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively, relative to a fixed origin O, as shown in Figure 1.

It is given that

$$\mathbf{a} = \mathbf{i} + \mathbf{j}, \quad \mathbf{b} = 3\mathbf{i} - \mathbf{j} + \mathbf{k} \quad \text{and} \quad \mathbf{c} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}\,.$$

Calculate

(*a*) $\mathbf{b} \times \mathbf{c}$,

(3)

(*b*) $\mathbf{a} \,.\, (\mathbf{b} \times \mathbf{c})$,

(2)

(*c*) the area of triangle OBC,

(2)

(*d*) the volume of the tetrahedron $OABC$.

(1)

3.

$$\mathbf{M} = \begin{pmatrix} 6 & 1 & -1 \\ 0 & 7 & 0 \\ 3 & -1 & 2 \end{pmatrix}$$

(*a*) Show that 7 is an eigenvalue of the matrix **M** and find the other two eigenvalues of **M**.

(5)

(*b*) Find an eigenvector corresponding to the eigenvalue 7.

(4)

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4. Given that $y = \text{arsinh}\,(\sqrt{x}),\quad x > 0,$

(*a*) find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, giving your answer as a simplified fraction.

(3)

(*b*) Hence, or otherwise, find

$$\int_{\frac{1}{4}}^{4} \frac{1}{\sqrt{[x(x+1)]}}\,\mathrm{d}x,$$

giving your answer in the form $\ln\left(\dfrac{a+b\sqrt{5}}{2}\right)$, where a and b are integers.

(6)

5.

$$I_n = \int_0^5 \frac{x^n}{\sqrt{(25-x^2)}}\,\mathrm{d}x, \quad n \geq 0.$$

(*a*) Find an expression for $\displaystyle\int \frac{x}{\sqrt{(25-x^2)}}\,\mathrm{d}x, \quad 0 \leq x \leq 5.$

(2)

(*b*) Using your answer to part (*a*), or otherwise, show that

$$I_n = \frac{25(n-1)}{n} I_{n-2}, \quad n \geq 2.$$

(5)

(*c*) Find I_4 in the form $k\pi$, where k is a fraction.

(4)

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6. The hyperbola H has equation $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, where a and b are constants.

The line L has equation $y = mx + c$, where m and c are constants.

(*a*) Given that L and H meet, show that the x-coordinates of the points of intersection are the roots of the equation

$$(a^2m^2 - b^2)x^2 + 2a^2mcx + a^2(c^2 + b^2) = 0.$$

(2)

Hence, given that L is a tangent to H,

(*b*) show that $a^2m^2 = b^2 + c^2$.

(2)

The hyperbola H' has equation $\dfrac{x^2}{25} - \dfrac{y^2}{16} = 1$.

(*c*) Find the equations of the tangents to H' which pass through the point (1, 4).

(7)

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7. The lines l_1 and l_2 have equations

$$\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} \alpha \\ -4 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}.$$

If the lines l_1 and l_2 intersect, find

(*a*) the value of α,

(4)

(*b*) an equation for the plane containing the lines l_1 and l_2, giving your answer in the form $ax + by + cz + d = 0$, where a, b, c and d are constants.

(4)

For other values of α, the lines l_1 and l_2 do not intersect and are skew lines.

Given that $\alpha = 2$,

(*c*) find the shortest distance between the lines l_1 and l_2 .

(3)

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8. A curve, which is part of an ellipse, has parametric equations

$$x = 3\cos\theta, \quad y = 5\sin\theta, \quad 0 \le \theta \le \frac{\pi}{2}.$$

The curve is rotated through 2π radians about the x-axis.

(*a*) Show that the area of the surface generated is given by the integral

$$k\pi\int_0^{\alpha} \sqrt{(16c^2 + 9)}\ \mathrm{d}c\,, \text{ where } c = \cos\theta,$$

and where k and α are constants to be found.

(6)

(*b*) Using the substitution $c = \frac{3}{4}\sinh u$, or otherwise, evaluate the integral, showing all of your working and giving the final answer to 3 significant figures.

(5)

TOTAL FOR PAPER: 75 MARKS

END

June 2009
6669 Further Pure Mathematics FP3 (new)
Mark Scheme

Question Number	Scheme	Marks
Q1	$\frac{7}{\cosh x}-\frac{\sinh x}{\cosh x}=5 \;\Rightarrow\; \frac{14}{e^x+e^{-x}}-\frac{(e^x-e^{-x})}{e^x+e^{-x}}=5$	M1
	$\therefore 14-(e^x-e^{-x})=5(e^x+e^{-x}) \;\Rightarrow\; 6e^x-14+4e^{-x}=0$	A1
	$\therefore 3e^{2x}-7e^x+2=0 \;\Rightarrow\; (3e^x-1)(e^x-2)=0$	M1
	$\therefore e^x=\frac{1}{3}$ or 2	A1
	$x=\ln(\frac{1}{3})$ or $\ln 2$	B1ft **[5]**
Alternative (i)	Write $7-\sinh x=5\cosh x$, then use exponential substitution $7-\frac{1}{2}(e^x-e^{-x})=\frac{5}{2}(e^x+e^{-x})$ Then proceed as method above.	M1
Alternative (ii)	$(7\text{sech}x-5)^2=\tanh^2 x=1-\text{sech}^2 x$	M1
	$50\text{sech}^2 x-70\text{sech}x+24=0$	A1
	$2(5\text{sech}x-3)(5\text{sech}x-4)=0$	M1
	$\text{sech}x=\frac{3}{5}$ or $\text{sech}x=\frac{4}{5}$	A1
	$x=\ln(\frac{1}{3})$ or $\ln 2$	B1ft
Q2 (a)	$\mathbf{b}\times\mathbf{c}=0\mathbf{i}+5\mathbf{j}+5\mathbf{k}$	M1 A1 A1 (3)
(b)	$\mathbf{a}.(\mathbf{b}\times\mathbf{c})=0+5=5$	M1 A1 ft (2)
(c)	Area of triangle $OBC=\frac{1}{2}\lvert 5\mathbf{j}+5\mathbf{k}\rvert=\frac{5}{2}\sqrt{2}$	M1 A1 (2)
(d)	Volume of tetrahedron $=\frac{1}{6}\times 5=\frac{5}{6}$	B1 ft (1) **[8]**

Question Number	Scheme	Marks
Q3 (a)	$\begin{vmatrix} 6-\lambda & 1 & -1 \\ 0 & 7-\lambda & 0 \\ 3 & -1 & 2-\lambda \end{vmatrix} = 0 \quad \therefore (6-\lambda)(7-\lambda)(2-\lambda)+3(7-\lambda)=0$	M1
	$(7-\lambda)=0$ verifies $\lambda=7$ is an eigenvalue (can be seen anywhere)	M1
	$\therefore (7-\lambda)\left\{12-8\lambda+\lambda^2+3\right\}=0 \quad \therefore (7-\lambda)\left\{\lambda^2-8\lambda+15\right\}=0$	A1
	$\therefore (7-\lambda)(\lambda-5)(\lambda-3)=0$ and 3 and 5 are the other two eigenvalues	M1 A1
		(5)
(b)	Set $\begin{pmatrix} 6 & 1 & -1 \\ 0 & 7 & 0 \\ 3 & -1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 7\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ or $\begin{pmatrix} -1 & 1 & -1 \\ 0 & 0 & 0 \\ 3 & -1 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	M1
	Solve $-x+y-z=0$ and $3x-y-5z=0$ to obtain $x = 3z$ or $y = 4z$ and a second equation which can contain 3 variables	M1 A1
	Obtain eigenvector as 3**i** + 4**j** + **k** (or multiple)	A1
		(4)
		[9]

Question Number	Scheme	Marks
Q4 (a)	$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} \times \frac{1}{\sqrt{1+(\sqrt{x})^2}}$	B1, M1
	$\therefore \frac{dy}{dx} = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\sqrt{1+x}} \quad \left(= \frac{1}{2\sqrt{x(1+x)}} \right)$	A1 (3)
(b)	$\therefore \int_{\frac{1}{4}}^{4} \frac{1}{\sqrt{x(x+1)}} dx = \left[2\text{ar}\sinh\sqrt{x} \right]_{\frac{1}{4}}^{4}$	M1
	$= \left[2\text{ar}\sinh 2 - 2\text{ar}\sinh(\frac{1}{2}) \right]$	M1
	$= \left[2\ln(2+\sqrt{5}) \right] - \left[2\ln(\frac{1}{2}+\sqrt{\frac{5}{4}}) \right]$	M1
	$2\ln\frac{(2+\sqrt{5})}{(\frac{1}{2}+\sqrt{(\frac{5}{4})})} = 2\ln\frac{2(2+\sqrt{5})}{(1+\sqrt{5})} = 2\ln\frac{2(\sqrt{5}+2)(\sqrt{5}-1)}{(\sqrt{5}+1)(\sqrt{5}-1)} = 2\ln\frac{(3+\sqrt{5})}{2}$	M1
	$= \ln\frac{(3+\sqrt{5})(3+\sqrt{5})}{4} = \ln\frac{14+6\sqrt{5}}{4} = \ln\left(\frac{7}{2}+\frac{3\sqrt{5}}{2}\right)$	A1 A1 (6) **[9]**
Alternative (i) for part (a)	Use $\sinh y = \sqrt{x}$ and state $\cosh y \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$	B1
	$\therefore \frac{dy}{dx} = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\sqrt{1+\sinh^2 y}} = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\sqrt{\left(1+\left(\sqrt{x}\right)^2\right)}}$	M1
	$\therefore \frac{dy}{dx} = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\sqrt{1+x}} \quad \left(= \frac{1}{2\sqrt{x(1+x)}} \right)$	A1 (3)
Alternative (i) for part (b)	Use $x = \tan^2\theta$, $\frac{dx}{d\theta} = 2\tan\theta\sec^2\theta$ to give $2\int\sec\theta d\theta = \left[2\ln(\sec\theta+\tan\theta)\right]$	M1
	$= \left[2\ln(\sec\theta+\tan\theta)\right]_{\tan\theta=\frac{1}{2}}^{\tan\theta=2}$ i.e. use of limits then proceed as before from line 3 of scheme	M1
Alternative (ii) for part (b)	Use $\int \frac{1}{\sqrt{[(x+\frac{1}{2})^2-\frac{1}{4}]}} dx = \text{arcosh}\frac{x+\frac{1}{2}}{\frac{1}{2}}$	M1
	$= \left[\text{arcosh}\,9 - \text{arcosh}(\frac{3}{2})\right]$	M1
	$= \left[\ln(9+\sqrt{80})\right] - \left[\ln(\frac{3}{2}+\frac{1}{2}\sqrt{5})\right]$	M1
	$= \ln\frac{(9+\sqrt{80})}{(\frac{3}{2}+\frac{1}{2}\sqrt{5})} = \ln\frac{2(9+\sqrt{80})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$,	M1
	$= \ln\frac{2(9+4\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})} = \ln\left(\frac{7}{2}+\frac{3\sqrt{5}}{2}\right)$	A1 A1 (6) **[9]**

Question Number	Scheme	Marks
Q5 (a)	$-(25-x^2)^{\frac{1}{2}}$ (+c)	M1A1 (2)
(b)	$I_n = \int x^{n-1}.\frac{x}{\sqrt{(25-x^2)}}\text{d}x = -x^{n-1}\sqrt{25-x^2} + \int (n-1)x^{n-2}\sqrt{(25-x^2}\text{d}x$	M1 A1ft
	$I_n = \left[-x^{n-1}\sqrt{25-x^2}\right]_0^5 + \int_0^5 \frac{(n-1)x^{n-2}(25-x^2)}{\sqrt{(25-x^2)}}\text{d}x$	M1
	$I_n = 0 + 25(n\text{-}1)\ I_{n-2}\ \text{-}\ (n\text{-}1)\ I_n$	M1
	$\therefore nI_n = 25(n-1)I_{n-2}$ and so $I_n = \frac{25(n-1)}{n}I_{n-2}$ ✳	A1 (5)
(c)	$I_0 = \int_0^5 \frac{1}{\sqrt{(25-x^2}}\text{d}x = \left[\arcsin(\frac{x}{5})\right]_0^5 = \frac{\pi}{2}$	M1 A1
	$I_4 = \frac{25\times 3}{4}\times\frac{25\times 1}{2}I_0 = \frac{1875}{16}\pi$	M1 A1 (4)
		[11]
Alternative for (b)	Using substitution $x = 5\sin\theta$	
	$I_n = 5^n\int_0^{\frac{\pi}{2}}\sin^n\theta\text{d}\theta = \left[-5^n\sin^{n-1}\theta\cos\theta\right]_0^{\frac{\pi}{2}} + 5^n(n-1)\int_0^{\frac{\pi}{2}}\sin^{n-2}\theta\cos^2\theta\text{d}\theta$	M1A1
	$= \left[-5^n\sin^{n-1}\theta\cos\theta\right]_0^{\frac{\pi}{2}} + 5^n(n-1)\int_0^{\frac{\pi}{2}}\sin^{n-2}\theta(1-\sin^2\theta)\text{d}\theta$	M1
	$I_n = 0 + 25(n\text{-}1)\ I_{n-2}\ \text{-}\ (n\text{-}1)\ I_n$	M1
	$\therefore nI_n = 25(n-1)I_{n-2}$ and so $I_n = \frac{25(n-1)}{n}I_{n-2}$ ✳	A1
	(need to see that $I_{n-2} = 5^{n-2}\int_0^{\frac{\pi}{2}}\sin^{n-2}\theta\text{d}\theta$ for final A1)	(5)

Question Number	Scheme	Marks
Q6 (a)	$\dfrac{x^2}{a^2}-\dfrac{(mx+c)^2}{b^2}=1$ and so $b^2x^2-a^2(mx+c)^2=a^2b^2$	M1
	$\therefore\ (b^2-a^2m^2)x^2-2a^2mcx-a^2(c^2+b^2)=0$	
	Or $(a^2m^2-b^2)x^2+2a^2mcx+a^2(c^2+b^2)=0$ ✳	A1 (2)
(b)	$(2a^2mc)^2=4(a^2m^2-b^2)\times a^2(c^2+b^2)$	M1
	$4a^4m^2c^2=-4a^2(b^2c^2+b^4-a^2m^2c^2-a^2m^2b^2)$	
	$c^2=a^2m^2-b^2$ or $a^2m^2=b^2+c^2$ ✳	A1 (2)
(c)	Substitute (1, 4) into $y=mx+c$ to give $4=m+c$ and	
	Substitute $a=5$ and $b=4$ into $c^2=a^2m^2-b^2$ to give $c^2=25m^2-16$	B1
	Solve simultaneous equations to eliminate m or c : $(4-m)^2=25m^2-16$	M1
	To obtain $24m^2+8m-32=0$	A1
	Solve to obtain $8(3m+4)(m-1)=0......m=..\text{or}...$	M1
	$m=1$ or $-\dfrac{4}{3}$	A1
	Substitute to get $c=3$ or $\dfrac{16}{3}$	M1
	Lines are $y=x+3$ and $3y+4x=16$	A1 (7) **[11]**

Question Number	Scheme	Marks
Q7 (a)	If the lines meet, $-1+3\lambda=-4+3\mu$ and $2+4\lambda=2\mu$	M1
	Solve to give $\lambda=0$ $\left(\mu=1\text{ but this need not be seen}\right)$.	M1 A1
	Also $1-\lambda=\alpha$ and so $\alpha=1$.	B1 (4)
(b)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 3 & 4 \\ 0 & 3 & 2 \end{vmatrix}$ = -6**i**+2**j**-3**k** is perpendicular to both lines and hence to the plane	M1 A1
	The plane has equation **r.n**=**a.n ,** which is –6x + 2y - 3z = -14,	M1
	i.e. –6x + 2y - 3z +14 = 0.	A1 o.a.e. (4)
OR (b)	**Alternative scheme**	
	Use (1, -1,2) and (α ,-4,0) in equation ax+by+cz+d=0	M1
	And third point so three equations, and attempt to solve	M1
	Obtain $6x-2y+3z=$	A1
	$(6x-2y+3z)-14=0$	A1 o.a.e. (4)
(c)	$(\mathbf{a}_1-\mathbf{a}_2)=\mathbf{i}-3\mathbf{j}-2\mathbf{k}$	M1
	Use formula $\dfrac{(\mathbf{a}_1-\mathbf{a}_2)\bullet\mathbf{n}}{\lvert\mathbf{n}\rvert}=\dfrac{(\mathbf{i}-3\mathbf{j}-2\mathbf{k})\bullet(-6\mathbf{i}+2\mathbf{j}-3\mathbf{k})}{\sqrt{(36+4+9)}}=\left(\dfrac{-6}{7}\right)$	M1
	Distance is $\dfrac{6}{7}$	A1 (3) **[11]**

Question Number	Scheme	Marks
Q8 (a)	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = -3\sin\theta,\ \frac{\mathrm{d}y}{\mathrm{d}\theta} = 5\cos\theta$	B1
	so S = $2\pi\int 5\sin\theta\sqrt{(-3\sin\theta)^2+(5\cos\theta)^2}\,\mathrm{d}\theta$	M1
	$\therefore S = \quad 2\pi\int 5\sin\theta\sqrt{9-9\cos^2\theta+25\cos^2\theta}\,\mathrm{d}\theta$	M1
	Let $c=\cos\theta$, $\frac{dc}{d\theta} = -\sin\theta$, limits 0 and $\frac{\pi}{2}$ become 1 and 0	M1
	So $S = k\pi\int_0^{\alpha}\sqrt{16c^2+9}\,\mathrm{d}c$, where $k = 10$, and α is 1	A1, A1 (6)
(b)	Let $c = \frac{3}{4}\sinh u$. Then $\frac{dc}{du} = \frac{3}{4}\cosh u$	M1
	So $S = k\pi\int_?^?\sqrt{9\sinh^2 u+9}\ \frac{3}{4}\cosh u\mathrm{d}u$	A1
	$= k\pi\int_?^?\ \frac{9}{4}\cosh^2 u\,\mathrm{d}u = k\pi\int_?^?\ \frac{9}{8}(\cosh 2u+1)\mathrm{d}u$	M1
	$= k\pi\left[\frac{9}{16}\sinh 2u+\frac{9}{8}u\right]_0^d$	A1
	$= \frac{45\pi}{4}\left[\frac{20}{9}+\ln 3\right]$ i.e. <u>117</u>	B1 (5) [11]